(1) $\dfrac{4-5i}{7i} \times \dfrac{-7i}{-7i} = \dfrac{-7i(4-5i)}{49}$

$$= \frac{-28i-35}{49} = \frac{-28}{49}i - \frac{35}{49}$$

$$= -\frac{4}{7}i - \frac{5}{7}$$

$$= \frac{5}{7} - \frac{4}{7}i$$

$$(5)\ \frac{10}{2+i} = x + yi$$

$$\frac{10}{(2+i)} \times \frac{2-i}{(2-i)} = \frac{10(2-i)}{2^2+1^2} = \frac{\cancel{10}^{\,2}(2-i)}{\cancel{5}}$$

$$2\times(2-i) = 4-2i$$

$$4-2i = x+yi$$

$$\boxed{x=4} \qquad \boxed{y=-2}$$

(4) $x^2 - y^2 + (x+y)i = 4i$

$$x^2 - y^2 + (x+y)i = 0 + 4i$$

$$x^2 - y^2 = 0$$

$$(x-y)(\cancel{x+y}) = 0$$

$$4(x-y) = 0$$

$$x - y = 0$$

$$x = y$$

$$\boxed{x + y = 4}$$

$$y + y = 4$$

$$2y = 4$$

$$\boxed{y = 2}$$

$$\boxed{x = 2}$$

Lesson(2)

Type of Two Roots

$$x^2-5x+6=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{5\pm\sqrt{(-5)^2-4\times1\times6}}{2\times1}$$

$$x=\frac{5\pm\sqrt{1}}{2}$$

$$x_1=\frac{5+1}{2} \qquad x_2=\frac{5-1}{2}$$

$$S.S=\{3, 2\}$$

Two different real roots

$$x^2+6x+9=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-6\pm\sqrt{(6)^2-4\times1\times9}}{2\times1}$$

$$x=\frac{-6\pm\sqrt{0}}{2}$$

$$x_2=-3 \qquad x_1=-3$$

$$S.S=\{-3\}$$

Two equal real roots

$$x^2-2x+4=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{2\pm\sqrt{(-2)^2-4\times1\times4}}{2}$$

$$x=\frac{2\pm\sqrt{-12}}{2}$$

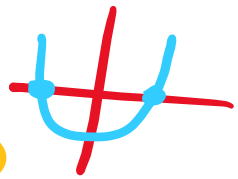
$$x=1\pm\sqrt{3}\,i$$

$$S.S=\{1+\sqrt{3}\,i,\ 1-\sqrt{3}\,i\}$$

Two complex non real roots

Discriminant $= b^2 - 4ac$

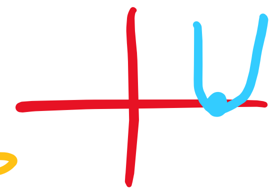
$+$



$$\boxed{b^2 - 4ac > 0}$$

Two different real roots

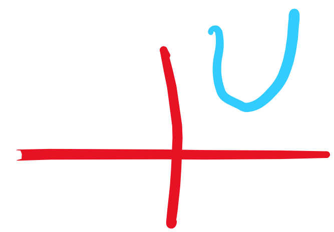
Zero



$$\boxed{b^2 - 4ac = 0}$$

Two equal real roots

$-$



$$\boxed{b^2 - 4ac < 0}$$

Two complex non real roots

- real root $\longrightarrow b^2 - 4ac \geq 0$
- Complex root $\longrightarrow b^2 - 4ac \leq 0$

Determine the type of two roots

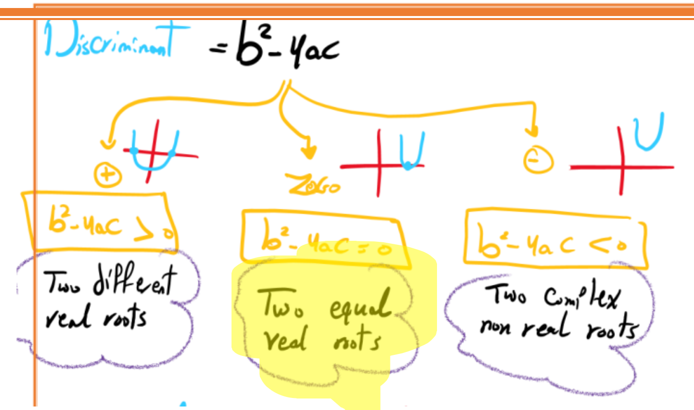
$\boxed{1}$ $x^2 - 6x + 9 = 0$

Soln

$a = 1$

$b = -6$

$c = 9$



$\text{Disc} = b^2 - 4ac$

$\text{Disc} = (-6)^2 - 4 \times 1 \times 6 =$ Zero

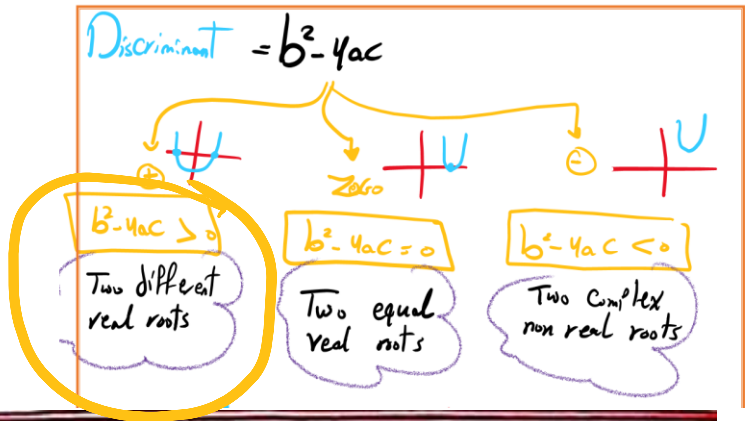
the Type : Two equal real roots

$\boxed{2}$ $2x^2+10x-5=0$

Soln

$b^2-4ac = 10^2-4\times2\times5 = \boxed{140}$

Type : Two different real root



$\boxed{3}$ $x^2-3x+5=0$

Soln

$b^2-4ac=(-3)^2-4\times1\times5$

$= -9-20 = \boxed{-29}$

Type : Two Complex non real roots

4 Determine the type of roots

$(x-11)-x(x-6)=0$

Soln

$x-11-x^2+6x=0$

$-x^2+7x-11=0$

$a=-1 \quad b=7 \quad \boxed{c=-11}$

$b^2-4ac=(7)^2-4\times-1\times-11=49-44=\boxed{5}$

$\boxed{\text{Two diff real roots}}$

Determine the type of roots

$$\frac{x}{x+1} + \frac{x}{x-1} = 3$$

a
b
c

Soln

$$\frac{x(x-1)+(x)(x+1)}{(x+1)(x-1)} = 3$$

$$\frac{x^2 - \cancel{x} + x^2 + \cancel{x}}{x^2-1} = 3$$

$$\frac{2x^2}{x^2-1} = \frac{3}{1}$$

① $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$

② $\frac{a}{b} + \frac{e}{f} = \frac{af+be}{bf}$

$$2x^2 = 3x^2 - 3$$

$$3x^2 - 2x^2 - 3 = 0$$

$$x^2 - 3 = 0$$

$\boxed{a=1}$ $\boxed{b=0}$ $\boxed{c=-3}$

$b^2 - 4ac = 0^2 - 4 \times 1 \times -3 = \boxed{12}$

Two different real roots

$\boxed{6}$ prove that the two roots ① of equation $7x^2 - 11x + 5 = 0$ are two Complex and non real roots and find the two roots

Soln

$a = 7 \quad b = -11 \quad c = 5$

$b^2 - 4ac = 49 - 4 \times 7 \times 5$

$= -19$

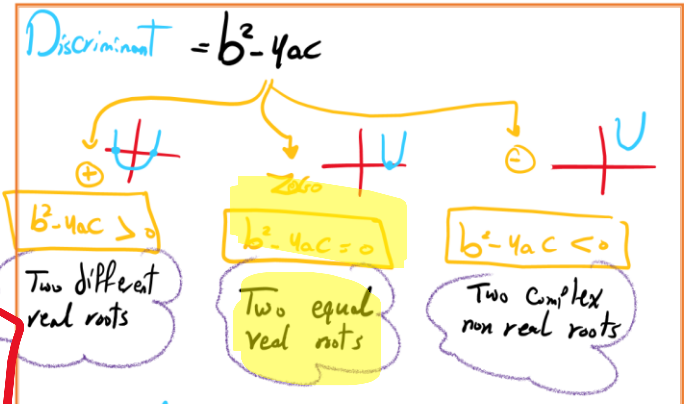
Two Complex non real roots

② $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{11 \pm \sqrt{19}\, i}{14}$

$S.S = \left\{ \frac{11 + \sqrt{19}\, i}{14} , \frac{11 - \sqrt{19}\, i}{14} \right\}$

7 Find the value of (k) which make the equation $2x^2-4x+k=0$ has two equal real roots

Soln $\boxed{a=2}$ $\boxed{b=-4}$ $\boxed{c=k}$



∴ Two equal real roots

∴ $b^2-4ac=0$

$16-4\times2\times k=0$

$16-8k=0$

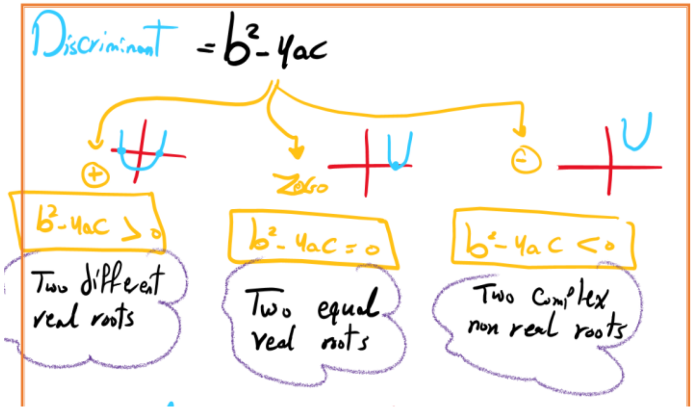
$8k=16$

$\boxed{k=2}$

8 if Two roots of equation

$$x^2 - \boxed{(k+4)}x + \boxed{(2k+5)} = 0$$

are equal ; Find Value of (K)



Soln

$a = 1$ $b = -(k+4)$ $c = (2k+5)$

$$b^2 - 4ac = 0$$

$$(k+4)^2 - 4 \times 1 \times (2k+5) = 0$$

$$(x+y)^2 = x^2 + 2xy + y^2$$

$$(x-y)^2 = x^2 - 2xy + y^2$$

$$k^2 + \cancel{8k} + 16 - \cancel{8k} - 20 = 0$$

$$k^2 - 4 = 0$$

$$k^2 = 4$$

$$\boxed{k = \pm 2}$$

9) Find the Value of (m) which satisfied that the equation

$x^2 - (2m-1)x + m^2 = 0$

has no real roots

Soln

$a = 1 \quad b = -(2m-1) \quad c = m^2$

no real roots

$b^2 - 4ac < 0$

$\geq$ closed

$>$ open

$(2m-1)^2 - \underline{4 \times 1 \times m^2} < 0$

$\cancel{4m^2} - 4m + 1 - \cancel{4m^2} < 0$

$-4m + 1 < 0$ ⊕

$1 < 4m$

$4m > 1$

$$\boxed{m > \frac{1}{4}}$$

$$\boxed{m \in \left]\frac{1}{4}, \infty\right[}$$

10 Find the value of (k) To Make this equation

$$x^2+2(k-1)x+k^2=0$$

has two real roots

$a=1 \quad b=2(k-1)=2k-2 \quad c=k^2$

Discriminant $=b^2-4ac$

$\oplus$	Zero	$\ominus$
$b^2-4ac>0$	$b^2-4ac=0$	$b^2-4ac<0$
Two different real roots	Two equal real roots	Two complex non real roots

∴ real root $\longrightarrow \boxed{b^2-4ac \geq 0}$

∴ Complex root $\longrightarrow b^2-4ac \leq 0$

$$b^2-4ac \geq 0$$

$$(2k-2)^2-4\times1\times k^2 \geq 0$$

$$\cancel{4k^2}-8k+4-\cancel{4k^2} \geq 0$$

$$-8k+4 \geq 0$$

$$-2k+1 \geq 0$$

$$-2k \geq -1$$

$$2k \leq 1$$

$$k \leq \frac{1}{2}$$

$$k \in \left]-\infty, \frac{1}{2}\right]$$

$$\boxed{\begin{array}{c} -2 < -1 \\ 2 > 1 \end{array}}$$

11 prove that the equation

$$(a-1)x^2 - ax + 1 = 0$$

has two real and different roots for all $a \in R - \{2\}$

$a^* = (a-1)$

$b^* = -a$

$c^* = 1$

Soln

$$b^{*2} - 4a^*c^* = (-a)^2 - 4(a-1)(1) =$$

$$= a^2 - 4a + 4 =$$

$$= (a-2)^2 > 0$$

$\therefore$ diff real root

$a \in \mathbb{R} - \{2\}$

12 prove that the two roots of eqn

$\frac{1}{x+a} = \frac{1}{x} + \frac{1}{a}$ are always not real if $a \in \mathbb{R}^*$; $x \notin [0, -a]$

Soln

$$\frac{1}{x+a} = \frac{a+x}{ax}$$

$$(x+a)^2 = ax$$

$$x^2 + 2ax + a^2 = ax$$

$$1x^2 + \boxed{a}x + \boxed{a^2} = 0$$

$a = 1 \qquad b = a \qquad c = a^2$

$$b^2 - 4ac = a^2 - 4 \times 1 \times a^2 = a^2 - 4a^2 = -3a^2 < 0$$

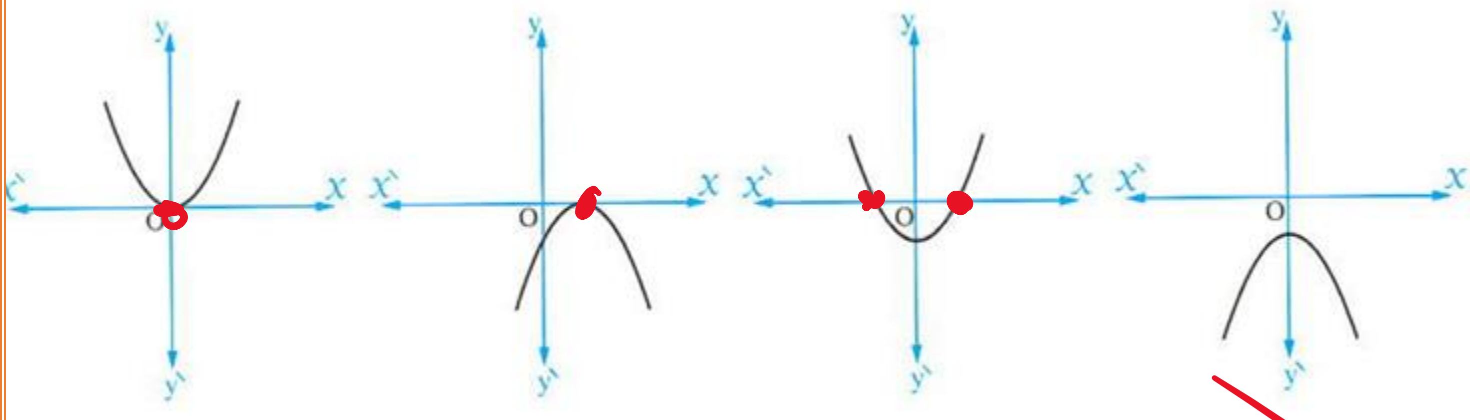
$b^2 - 4ac < 0$

∴ Two Complex non real roots

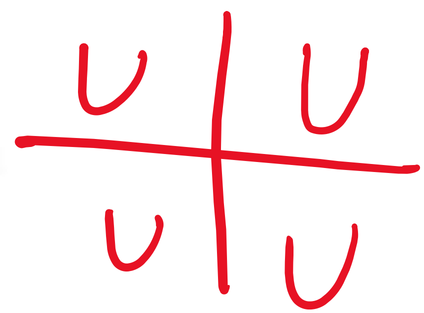
$a \in \mathbb{R}^*$

(21) In the quadratic equation $f(X) = 0$, if the discriminant is negative , then which of the following graphs is the graph of the function $f(X)$?

$b^2 - 4ac < 0$

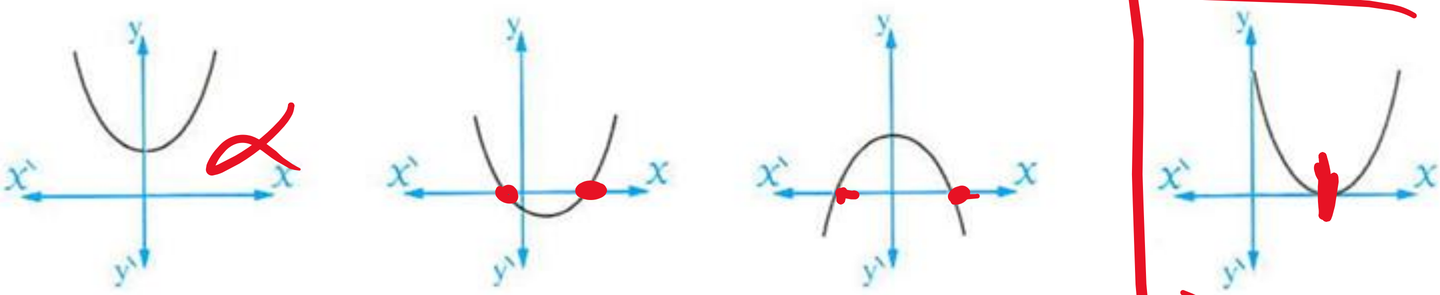


(a) (b) (c) (d)



(22) Each of the following figures represents the curve of the function f :

$f(X) = a X^2 + b X + c$ which of these figures does have $b^2 - 4ac = 0$



(a) (b) (c) (d)

Discriminant $= b^2 - 4ac$

$\oplus$ — $b^2 - 4ac > 0$ — Two different real roots

zero — $b^2 - 4ac = 0$ — Two equal real roots

$\ominus$ — $b^2 - 4ac < 0$ — Two complex non real roots

* real root $\longrightarrow b^2 - 4ac \geq 0$
* Complex root $\longrightarrow b^2 - 4ac \leq 0$

Find the interval to which a belongs that makes the two roots of the equation :

$(a + 2) X^2 + (2a + 3) X + a - 1 = 0$ real numbers.

(a + 2) x^2 + (2 a + 3) x + a – 1 = 0 real numbers.

$$b^2 - 4ac = (2a+3)^2 - 4 \times (a+2)(a-1)$$

$-a$

$2a$

$$4a^2 + 12a + 9 - 4\left[a^2 + a - 2\right]$$

$$\cancel{4a^2} + 12a + 9 - \cancel{4a^2} - 4a + 8$$

$$\boxed{8a + 17 = b^2 - 4ac}$$

$$8a + 17 \geq 0$$

$-$

$$8a \geq -17$$

$$a \geq \frac{-17}{8}$$

$a^* = (a+2)$

$b^* = (2a+3)$

$c^* = a - 1$

$$\boxed{a \in \left[\frac{-17}{8}, \infty\right[}$$

Rational roots :-

In quadratic equation

$ax^2 + bx + c = 0 \quad ; \quad a \neq 0$

Two roots are rational if ;

① $a, b, c \in \mathbb{Q}$

② $(b^2 - 4ac)$ is perfect square

$\sqrt{5}$

$\sqrt{Disc}$

$Disc = \{1, 4, 9, 16, 25, \text{-----}\}$

Rational roots

1. $a, b, c \in Q$
2. $b^2 - 4ac$ is p.s $= \{1, 4, 9, 16, 25, 36, \dots\}$

Q' → $\sqrt{5}$, $\sqrt{6}$, $\sqrt[3]{6}$, $\sqrt[3]{9}$, π, 2π, $3\pi + 1$

[13] Which of them has two rational roots, Show that without solving

(i) $x^2 + 3x + \sqrt{5} = 0$

Soln

$a = 1 \in Q$ $\quad b = 3 \in Q$ $\quad c = \sqrt{5} \notin Q$

has no rational Roots

(ii) $x^2+4x+3=0$

soln

$a=1$

$b=4$ $\in Q \longrightarrow$ (1)

$c=3$

$b^2-4ac = 16-4\times3\times1 = 16-12 = \boxed{4}$

b^2-4ac p.s $\longrightarrow$ (2)

from (1), (2)

the roots is Rational

(iii) $2(x+3) + x(x-1) = 9$

Soln

$$2x+6+x^2-x=9$$

$$x^2+x+6=9$$

$$x^2+x-3=0$$

① $a=1$, $b=1$, $c=-3$ $\in Q$

② $b^2-4ac = 1^2-4\times1\times-3 = 1+12 = \boxed{13}$ not p.s

from ①, ②

don't have rational roots

Prove that the two roots of the equation :

$x^2 + kx + k = 1$ are always rational where $k \in \mathbb{Q}$

$x^2 + kx + (k-1) = 0$

$\boxed{1}$ $a = 1 \in Q$

$b = k \in Q$

$c = k - 1 \in Q$

$\boxed{2}$ $b^2 - 4ac = (k)^2 - 4(k-1)$

$= k^2 - 4k + 4$

$= (k-2)^2$ is p.s

$\therefore$ Always rational roots

If L and M are two rational numbers , then prove that the two roots of the equation :

$L x^2 + (L - M) x - M = 0$ are rational numbers.

$a = L \in Q$

1 $b = L - M \in Q$

$c = -M \in Q$

2 $b^2 - 4ac = (L - M)^2 + 4LM$

$= L^2 - 2LM + M^2 + 4LM$

$= L^2 + 2LM + M^2$

$= (L + M)^2$ is P.S

∴ Two roots always Rational

a , b , c are rational numbers , $a \neq 0$, then the equation : $a x^2 + b x + c = 0$ has rational roots if $b^2 - 4ac =$ ....

(a) positive real number.

(b) negative real number.

~~(c) perfect square real number.~~

(d) zero.

1

2

If the roots of the equation $a x^2 + b x + c = 0$ where $a > 0$ are real and equal , then the roots of the equation $a x^2 + b x + c + 1 = 0$ are ....

(a) real and equal.

(b) real and different.

(c) complex and not real.

(d) rational.

$b^2 - 4ac = 0$

$a = a$

$b = b$

$c = c + 1$

$b^2 - 4ac$

$b^2 - 4a(c+1)$

$b^2 - 4ac - 4a$

$0 - 4a < 0$

$\ominus \times \oplus$

$$b^2-4ac$$

$+$

Two diff

p.s

$\boxed{\text{rational number}}$

$\boxed{0}$

Two equal

$-$

Two complx non real

$$b^2-4ac \geq 0$$

$$b^2-4ac \leq 0$$

$\boxed{\text{real}}$

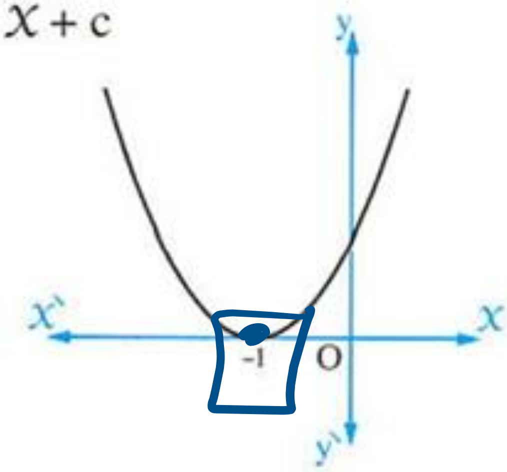
$\boxed{\text{Complex Conjugate}}$

The given figure represents the function $f : f(x) = a x^2 + b x + c$, then $(b^2 - 4ac) \times f(3) = ...$

$0 \times f(3) = 0$

(a) 3 (b) -1

(c) -3 (d) zero



$b^2 - 4ac = 0$